LINKS

Contour map  
<https://www.khanacademy.org/math/multivariable-calculus/thinking-about-multivariable-function/ways-to-represent-multivariable-functions/a/contour-maps>  
  
Vector field  
<https://www.khanacademy.org/math/multivariable-calculus/thinking-about-multivariable-function/ways-to-represent-multivariable-functions/a/vector-fields>  
  
Gradient (operator)   
<https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/partial-derivative-and-gradient-articles/a/the-gradient>

Exercises (gradient):  
<https://math.libretexts.org/Courses/Monroe_Community_College/MTH_212_Calculus_III/Chapter_13%3A_Functions_of_Multiple_Variables_and_Partial_Derivatives/13.6%3A_Directional_Derivatives_and_the_Gradient/13.6E%3A_Directional_Derivatives_and_the_Gradient_(Exercises)>

<https://openstax.org/books/calculus-volume-3/pages/4-6-directional-derivatives-and-the-gradient>

(course and exercises)

<https://www.siyavula.com/read/maths/grade-12/differential-calculus/06-differential-calculus-04>

[about derivatives, partial derivatives, tangent, gradient: you can focus on the second part of the site, with the exercises, also I don't have to focus on the way to find the vector normal to a plane]

LINKS  
From constrained to unconstrained problem : the Lagrangian function/lagrange multipliers (different names for the same method)  
<https://www.khanacademy.org/math/multivariable-calculus/applications-of-multivariable-derivatives/constrained-optimization/a/lagrange-multipliers-single-constraint>

(interesting reading, please try also to change the value of k in the graph with the contour line f(x,y)=k whenever it's possible)

<https://medium.com/@jonathan_hui/machine-learning-lagrange-multiplier-dual-decomposition-4afe66158c9>

(more specific/mathematical but nicely written,also goes deep into the topic: dual problem etc...)

Example of Lagrangian multipliers used to solve a business problem

<https://towardsdatascience.com/optimization-with-constraints-using-lagrange-multiplier-in-python-82769c9a43fe>

Reminder and exercises:

<https://tutorial.math.lamar.edu/classes/calciii/lagrangemultipliers.aspx>

Note: there is no link about the use of the extreme value theorem because in most problems with an equality constraint, you won't have to prove that part (you can see that in the videos below) so I won't ask you to use it.

VIDEOS (explanations and examples)

Lagrangian function (12min)

<https://youtu.be/hQ4UNu1P2kw>

Lagrange multipliers: explanation and example (12min)

<https://youtu.be/8mjcnxGMwFo>

Lagrange multiplier example with 3 variables (13min)

<https://youtu.be/nDuS5uQ7-lo>

Lagrange multiplier example (2 parts around 6min each)

part I  
<https://youtu.be/BSKtQcLQLWU>

part II  
<https://youtu.be/5gCx5YCmXbI>

- Lagrange multipliers with several equality constraints  
- Lagrange multipliers in the case of inequality constraints

LINKS  
Examples of Lagrange multipliers with 2 equalities (Example 4) or with 1 inequality (example 5)  
<https://tutorial.math.lamar.edu/Classes/CalcIII/LagrangeMultipliers.aspx>

Critical points (sometimes called stable points), Fermat's theorem, Saddle points and Second Partial Derivative Test (also includes a reminder about the second derivative test i.e. case of a single variable and a lot of exercises).

<https://openstax.org/books/calculus-volume-3/pages/4-7-maxima-minima-problems>

Second partial derivative test (a maybe more intuitive explanation of it)

<https://www.khanacademy.org/math/multivariable-calculus/applications-of-multivariable-derivatives/optimizing-multivariable-functions/a/second-partial-derivative-test>

Exercise:

<https://www.khanacademy.org/math/multivariable-calculus/applications-of-multivariable-derivatives/optimizing-multivariable-functions/a/examples-second-partial-derivative-test>

ARTICLES

KKT conditions explanation  
<https://ebrary.net/134820/mathematics/constrained_optimization_kuhn_tucker_conditions>

KKT conditions and inequality conditions (very visual explanation)  
<https://mjo.osborne.economics.utoronto.ca/index.php/tutorial/index/1/ktc/t>

VIDEOS

KKT conditions  
<https://youtu.be/HIm3Z0L90Co>

(geometric intuition)  
<https://youtu.be/2syXAny8PfE>

(theorem)

KKT conditions (explanations and a few videos, example 5 is off limits for us)  
<https://apmonitor.com/me575/index.php/Main/KuhnTucker>